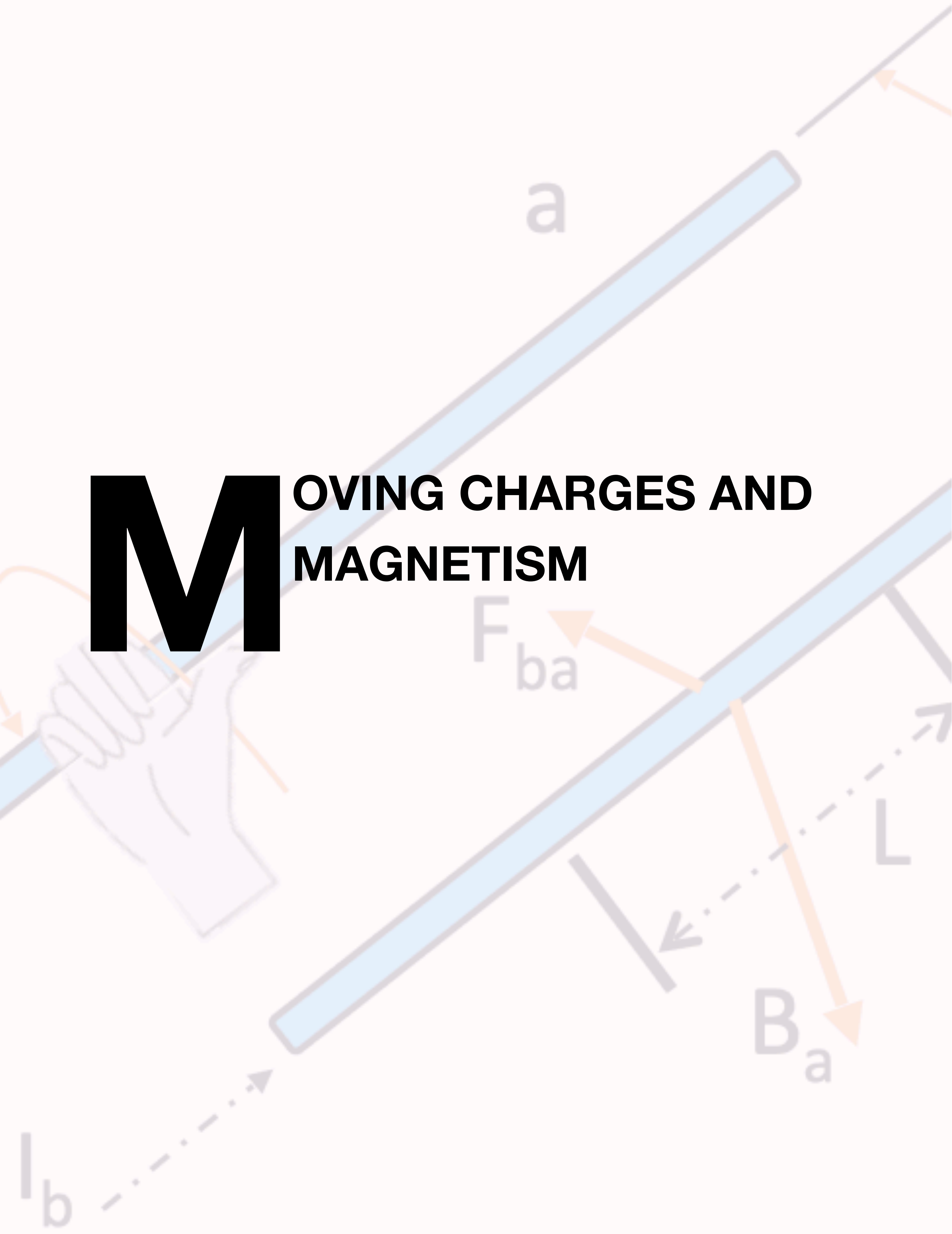
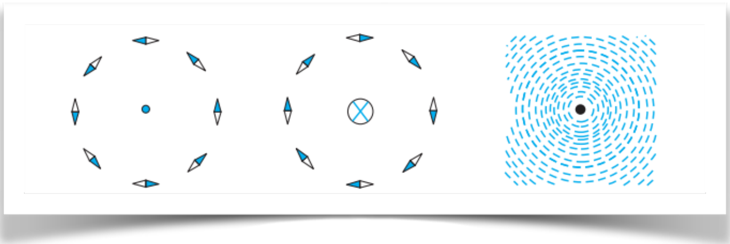
MOVING CHARGES AND MAGNETISM



INTRODUCTION

In 1820, Danish physicist Hans Christian Oersted noticed that a current causes deflection in a nearby compass needle.

He noticed that a current in a straight wire caused a noticeable deflection in a nearby magnetic compass needle.
Reversing the direction of the current reverses the orientation of the needle.
The deflection increases on increasing the current or bringing the needle closer to the wire. Iron filings sprinkled around the wire arrange themselves in concentric circles with the wire as the centre as shown below.



With his simple experiments, Oersted concluded that moving charges or currents produced a magnetic field in the surrounding space.
In 1864, the laws obeyed by electricity and magnetism were unified and formulated by James Maxwell.

SOURCES & FIELDS:

Just as static charges produce an electric field, the currents or moving charges produce (in addition) a magnetic field, denoted by B (r), again a vector field.

Principle of superposition:

The magnetic field of several sources is the vector addition of magnetic field of each individual source.

MAGNETIC FIELD & LORENTZ FORCE:

A point charge q (moving with a velocity v, located at r at a given time t) in presence of both the electric field E (r) and the magnetic field B (r).

The force on an electric charge q due to both of them can be written as (Lorentz Force)

$$F = q[\vec{E} + (\vec{v} \times \vec{B})] = \vec{F}_{electric} + \vec{F}_{magnetic}$$

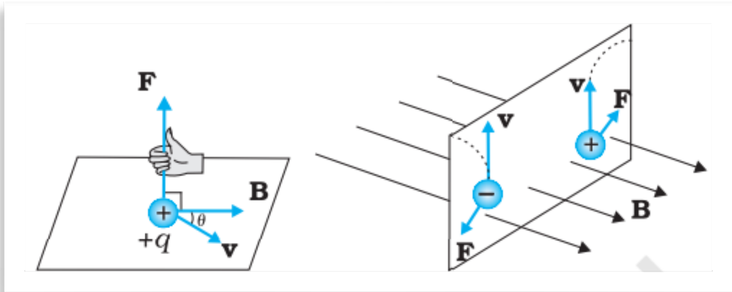
The Magnetic Force is,

$F_B = q[\vec{v} \times \vec{B}] = qvB\sin\theta\ \hat{n}$, where θ is the angle between **v** and **B**

Characteristics of Magnetic Force:

1. Force on a negative charge is opposite to that on a positive charge.
2. The magnetic force is zero if charge is not moving (as then |v|= 0)
3. Magnetic Force is zero if v and B are parallel or antiparallel.
4. The force acts in a (sideways) direction perpendicular to both the velocity and the magnetic field. Its direction is given by the screw rule or right hand rule for vector (or cross) product.

Right Hand Rule:



The Rt. Hand rule states that, if the right hand fingers are curled in the direction v to B then the direction in which the thumb is stretched, gives the direction of the magnetic force on a positive charge charge. The direction of force will be opposite in case of a negative charge.

SI Unit of Magnetic Field:

- T (Tesla)

We have the magnetic force given by, $F = q\ [v \times B] = qvB \sin\theta\ \hat{n}$

The magnitude of magnetic field *B* is 1 SI unit, when the force acting on a unit charge (1 C), moving perpendicular to **B** with a speed 1m/s, is one newton.

Dimensionally, we have [*B*] = [*F/qv*] i.e., Newton second / (coulomb metre) = tesla (T)

Smaller unit of magnetic field B is gauss (G) $= 10^{-4}T$

Note: The earth's magnetic field is about 3.6 × 10–5 T

Magnetic force on a current-carrying conductor :

Consider a rod of a uniform cross-sectional area *A* and length l.

Let the number density of electrons in it *n*.

Hence, the total number of free electrons is, $N = nlA$.

Drift velocity of charges be, v_d.

In the presence of an external magnetic field B, the force on these charges is,

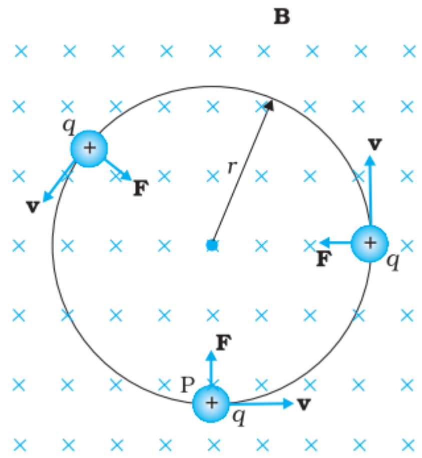
$F = q(\vec{V} \times \vec{B}) = (nlA)e(\vec{v_d} \times \vec{B}) = I(\vec{l} \times \vec{B})$

Where $\vec{l}$ is a vector of magnitude length, l with direction of current, I.

MOTION OF A CHARGE IN AN EXTERNAL MAGNETIC FIELD:

Consider a point charge q moving with a velocity v in an external uniform magnetic field B.

Case I: Charge, q moving perpendicular to the magnetic field B.



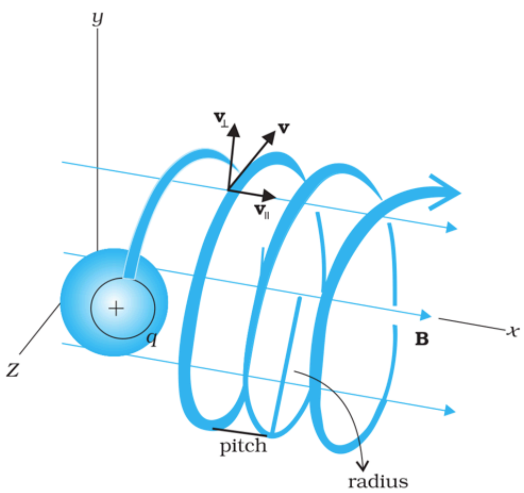
Here, the force, F=*q* (v × B)=qvB, is directed perpendicular to v and acts as a centripetal force and hence the charge will describe a circle when v and B are perpendicular.

Thus the centripetal force, $\dfrac{mv^2}{r} = qvB$

Which gives the radius of the circular path, $r = \dfrac{mv}{qB}$ ......(1)

Case II: Charge moving parallel to the magnetic field:

A charge moving parallel to the magnetic field doesn't experience any force and hence moves undeflected.



Case III: Charge, q moving with velocity components both parallel and perpendicular to magnetic field B.

Here, the velocity v has components both parallel to B (v_B) and perpendicular to B $(v_\perp)$ as shown in the diagram.

v_B is along the field which does not produce any force on the charge and makes the charge to go along the field itself. Whereas, $v_\perp$ is perpendicular to B and causes a magnetic forces which acts as centripetal force and make the charge to trace a circular path.

The straight path along B and a circular path perpendicular to B cause a resultant path which is spiral or helical motion as shown in the diagram.

Using eq (1), the radius of helix is, $r = \dfrac{mv_{\perp}}{qB}$……….(2)

If ω is the angular frequency, $\omega = 2\pi f = \dfrac{v}{r} = \dfrac{v}{(\frac{mv}{qB})} = \dfrac{qB}{m}$ which is independent of velocity or energy.

Time period of revolution is, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{(\frac{qB}{m})} = \dfrac{2\pi m}{qB}$……….(3)

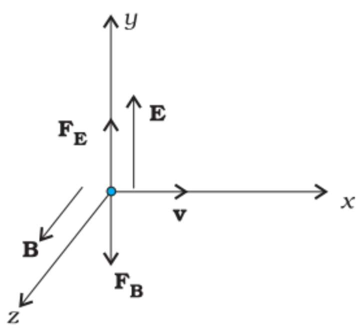
PITCH (*p*):

It is the distance moved along the magnetic field in one rotation. Given by,

$p = v_{||}T = v_B(\dfrac{2\pi m}{qB})$……….(4)

VELOCITY SELECTOR:

(Motion of a charge in a combined electric and magnetic fields)



Consider a point charge, q moving with a velocity, v in a direction perpendicular to both mutually perpendicular uniform electric and magnetic fields as shown in the diagram.

As we know, the Lorentz force on the charge is,

$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})] = \vec{F}_E + \vec{F}_B$ ……….(a)

Electric Force, $\vec{F}_E = q\vec{E} = qE\hat{j}$……….b)

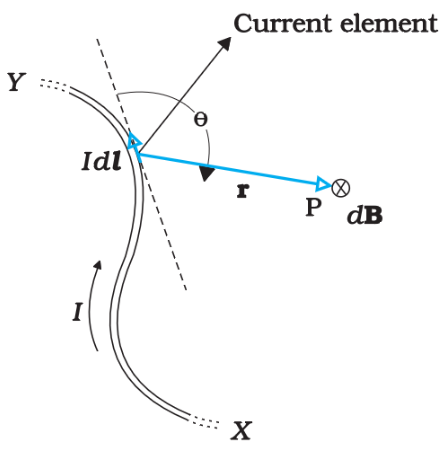
Magnetic force, $\vec{F}_B = q(\vec{v} \times \vec{B}) = q(v\hat{i} + B\hat{k}) = -qvB\hat{j}$ ……….(c)

sub (b) and (c) in (a), we get the net force, $\vec{F} = q(E - vB)\hat{j}$

The net force will be zero if $E = vB$ or $v = \dfrac{E}{B}$ and the charge will move undeflected in the fields.

This condition can be used to select charged particles of a particular velocity from a beam of charges of different velocities.
The crossed E and B fields serves as a **velocity selector**.

Magnetic Field due to a Current Element, Biot-Savart Law



Statement of Biot-Savart Law:
Magnetic field (dB) at a point due to a current element is directly proportional to the current through the current element (I) and its length (dl) and inversely proportional to the square of the distance between them (r). Its direction is perpendicular to the plane of dl and r.

i.e., $\overrightarrow{dB} \propto \frac{I(\overrightarrow{dl} \times \overrightarrow{r})}{r^3}$

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I(\overrightarrow{dl} \times \overrightarrow{r})}{r^3}$$

The direction of $\overrightarrow{dl} \times \vec{r}$ is given by Right Hand rule.

Where $\frac{\mu_0}{4\pi} = 10^{-7}$ Tm/A is the constant of proportionality in case of vacuum. And μ_0 is called the permeability of free space or vacuum.

Thus the magnitude of this field is $\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I(\overrightarrow{dl} \times \overrightarrow{r})}{r^3}$

Comparison of Biot-Savart Law of Magnetic Field and Coulomb's law of Electric Field:

<u>Similarities</u>

- Long range.
- Inversely proportional to square of the distance.
- Obey Principal of superposition.

Difference**s**

- Source of electric field is a scalar, charge (q)
- Source of magnetic field is a vector $I\vec{dl}$

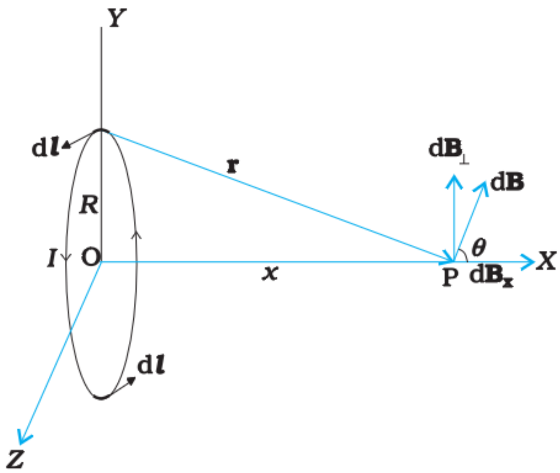
Relation between permeability and permittivity of free space:

$$\epsilon_0\mu_0 = (4\pi\epsilon_0)\left(\frac{\mu_0}{4\pi}\right) = \frac{1}{9\times10^9}\left(10^{-7}\right) = \frac{1}{\left(3\times10^8\right)^2} = \frac{1}{c^2}$$

In SI units, μ0 is fixed to be equal to 4π × 10–7 in magnitude.

Magnetic Field on the Axis of a Circular Current Loop

Consider a circular coil of radius R carrying current I as shown. Let P be a point on the axis of the coil at a distance of x from the centre O.



Let dl be a current element of the coil as shown which is distant r from P.

From basic principles, the current element dl will produce a magnetic $\vec{dB}$ at P directed as shown (perpendicular to both r and dl) given by,

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I(\vec{dl}\times\vec{r})}{r^3}$$

In magnitude,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,|\,\vec{dl}\times\vec{r}\,|}{r^3} \quad ....\ (1)$$

$|\,\vec{dl}\times\vec{r}\,| = r\ dl\sin(90) = r\ dl$ ----- since r is perpendicular to dl.

Eq. (1) gives,

$$dB = \frac{\mu_0}{4\pi}\frac{I\ r\ dl}{r^3} = \frac{\mu_0}{4\pi}\frac{I\ dl}{r^2} \quad ....\ (2)$$

This $\vec{dB}$ can be resolved into two perpendicular components $\vec{dB_\perp}$ and $\vec{dB_x}$ such that,

$\vec{dB_x} = dB_x$

When the components perpendicular to the x-axis are summed over, they cancel out and we obtain a null result. For example, the $\overrightarrow{dB_\perp}$ component due to dl is cancelled by the contribution due to the diametrically opposite dl element. Whereas the dB_x components are summed over along the axis.

From the diagram, $\cos\theta = \dfrac{R}{r}$

that gives us, $dB_x = dB\cos\theta = \left(\dfrac{\mu_0}{4\pi}\dfrac{I\,dl}{r^2}\right)\left(\dfrac{R}{r}\right) = \dfrac{\mu_0}{4\pi}\dfrac{I\,dl\,R}{r^3}$

But, $r = \sqrt{(x^2+R^2)} = (x^2+R^2)^{\frac{1}{2}}$

$$\therefore dB_x = \frac{\mu_0}{4\pi}\frac{I\,dl\,R}{(x^2+R^2)^{\frac{3}{2}}}$$

Thus the net magnetic field at P is the integration of dB_x over the complete loop.

I.e., $B = \sum dB_x = \sum \dfrac{\mu_0}{4\pi}\dfrac{I\,dl\,R}{(x^2+R^2)^{\frac{3}{2}}} = \dfrac{\mu_0}{4\pi}\dfrac{I\,\sum dl\,R}{(x^2+R^2)^{\frac{3}{2}}} = \dfrac{\mu_0}{4\pi}\dfrac{I\,(2\pi R)\,R}{(x^2+R^2)^{\frac{3}{2}}}$

$$\therefore B = \frac{\mu_0 I\,R^2}{2\,(x^2+R^2)^{\frac{3}{2}}} \quad \text{..........(3)}$$

SPECIAL CASE: FIELD AT THE CENTRE OF THE LOOP (COIL)

Here $x = 0$, from eq. (3) we get,

$$B = \frac{\mu_0 I\,R^2}{2\,(x^2+R^2)^{\frac{3}{2}}} = \frac{\mu_0 I\,R^2}{2\,(0+R^2)^{\frac{3}{2}}} = \frac{\mu_0 I\,R^2}{2R^3}$$

$\therefore B = \dfrac{\mu_0 I}{2R}$ .... (4) is the magnetic field at the centre of the loop.

The direction of the magnetic field is given by (another) right-hand thumb rule

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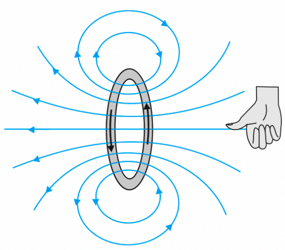
Right-hand thumb rule:

Curl the palm of your right hand around the circular wire with the fingers pointing in the direction of the current. The right-hand thumb gives the direction of the magnetic field.



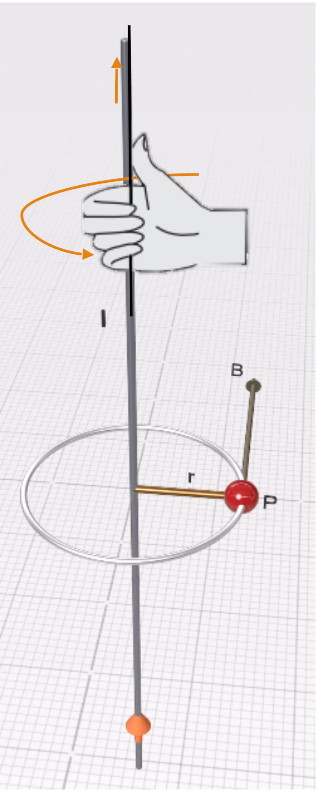
Ampere's Circuital Law

It states that the $\oint \vec{B} \cdot \vec{dl}$ of the resultant magnetic field along a closed, plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve called amperian loop.

i.e., $\oint \vec{B} \cdot \vec{dl} = \mu_0 \, I_e$ ....here I_e is the total current enclosed by the loop.

APPLICATIONS OF AMPERE'S CIRCUITAL LAW-

Magnetic field of an infinite straight conductor carrying conductor:



Consider an infinite straight conductor carrying current I. Let P be a point at a perpendicular distance r from the conductor.
In this case the amperian loop can be an imaginary circle with conductor as is perpendicular axis and P on its circumference.

The magnetic field at any point on the circumference of this loop is constant (B) and tangential to the loop.

Applying Ampere's Circuital Law to the loop, we get,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \, I_e$$

i.e., $\oint B \, dl \cos\theta = \mu_0 \, I$

i.e., $B \sum dl \cos\theta = \mu_0 \, I$

Or $B \, (2\pi r)(\cos 0) = \mu_0 \, I$; $\theta = 0$ as B is tangential.

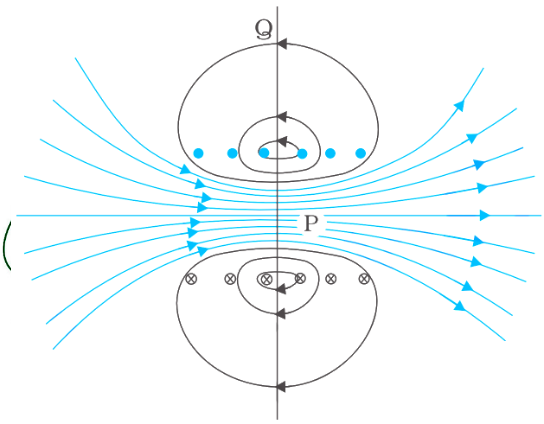
$$\therefore B = \frac{\mu_0 \, I}{2\pi r}$$

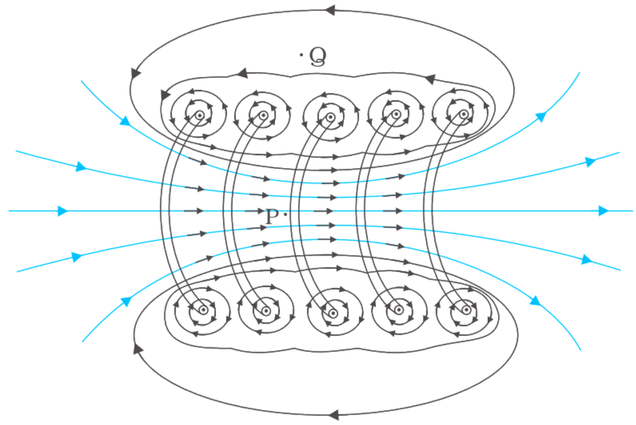
Direction of magnetic field around the conductor is given by the ***right-hand rule***: *Grasp the wire in your right hand with your extended thumb pointing in the direction of the current. Your fingers will curl around in the direction of the magnetic field.*

The Solenoid:

Solenoid is a spring like helical coil wound closely on an insulated cylinder.

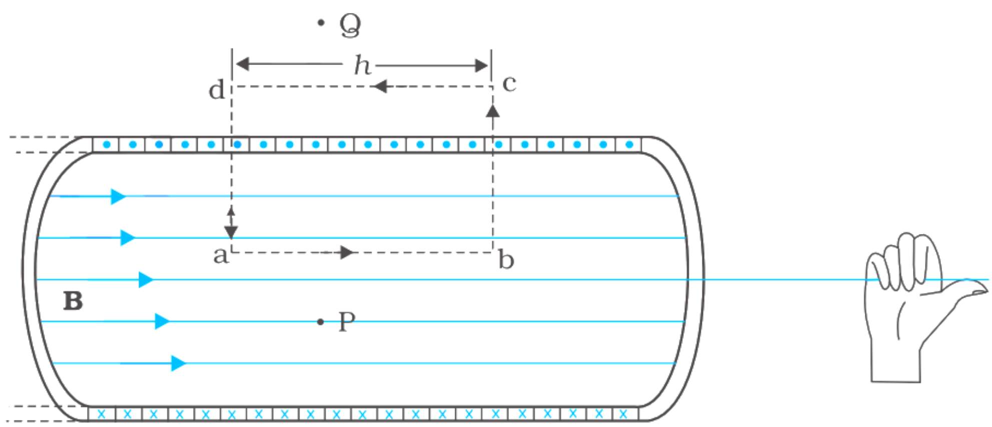
Consider a long solenoid (length is large compared to its radius) carrying current.
Magnetic Field produced by a such a solenoid is as shown in the picture.





MAGNETIC FIELD OF A SOLENOID:

Let us now consider a cross section of this solenoid.
The magnetic field in the middle of the solenoid (at P) is almost uniform, strong and along the axis. Whereas, in the exterior mid region (at Q), the field is very weak but along the axis as well. For a long solenoid, the field outside the solenoid approaches to zero.



Consider a rectangular amperian loop abcd as shown above.
The field is zero along cd (exterior region)
The field is zero along transverse sections bc and da.

Thus the relevant section is only ab. Let B be the field along ab of length ‘h’.

Let n be the number of turns per unit length, then the total number of turns in the length h is nh.

The current enclosed by the amperian loop is, $I_e = NI = nhI$

Thus, applying Ampere's Circuital Law to the loop abcd we get,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \, I_e$$

i.e., $\oint_{ab} \vec{B} \cdot \vec{dl} + \oint_{bc} \vec{B} \cdot \vec{dl} + \oint_{cd} \vec{B} \cdot \vec{dl} + \oint_{da} \vec{B} \cdot \vec{dl} = \mu_0 \, I_e$

i.e., $\oint_{ab} \vec{B} \cdot \vec{dl} + 0 + 0 + 0 = \mu_0 \, (nhI)$

i.e., $B \sum dl \cos\theta = \mu_0 \, (nhI)$

i.e., $B \; (h)(\cos 0) = \mu_0 \, (nhI)$, since B and h are parallel in the middle region.

$$\therefore B = \mu_0 \, nI$$

The direction of the field is given by the right-hand rule.

NOTE: The magnetic field at the ends of a solenoid is half of that at its middle. i.e., $B = \frac{\mu_0 \, nI}{2}$

Force between Two Parallel Currents, the Ampere

Every conductor carrying current produces magnetic field and in such a magnetic an another current carrying conductor always experiences a magnetic force.

Consider two long parallel conductors a and b carrying currents Ia and Ib separated by a distance d.

Current Ia of conductor a produces a magnetic field Ba at any point along the conductor b given by,

$B_a = \dfrac{\mu_0 I_a}{2\pi d}$ ....(1)

In this magnetic field, the conductor b experiences a force towards a. The force on a segment of L of b due to a is,

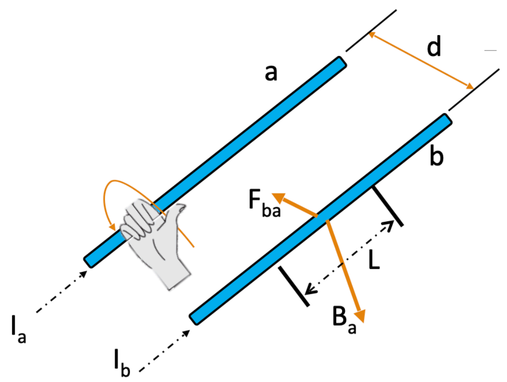
$$\vec{F} = I(\vec{L} \times \vec{B})$$

$$\implies F_{ba} = I_b L B_a \sin(90) = I_b \left(\frac{\mu_0 I_a}{2\pi d} \right) L$$

i.e., $F_{ba} = \dfrac{\mu_0 I_a I_b}{2\pi d} L$ ....(2)

Hence the force per unit length is, $f_{ba} = \dfrac{F_{ba}}{L}$

$\implies f_{ba} = \dfrac{F_{ba}}{L} = \dfrac{\mu_0 I_a I_b}{2\pi d}$...........(3)



Similarly, we may find the force on a due to b as

$F_{ab} = -F_{ba}$

This is consistent with Newton's third law.

Using right hand rule it can be shown that currents flowing in the same direction attract each other and oppositely directed currents repel each other.

"Parallel currents attract and antiparallel currents repel".

Magnetic Moment of a current

Magnetic moment of a current loop is given by the product of its area and the current through it.

i.e., $\vec{m} = I\vec{A}$ ……………. SI Unit is Am²

For N number of turns in the loop, the magnetic moment is $\vec{m} = NI\vec{A}$

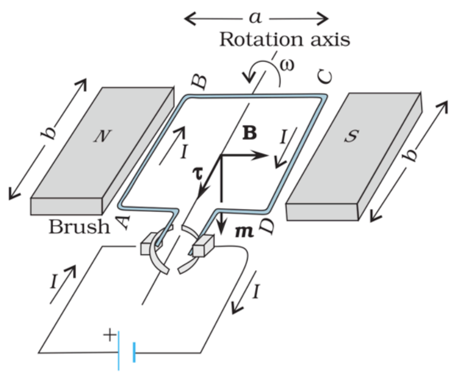
Torque on a rectangular current loop in a uniform magnetic field

Consider a rectangular loop ABCD carrying current I is placed in an uniform magnetic field B as shown in the diagram such that the field is in the plane of the loop.

The Magnetic does not exert any force on the arms AD and BC as they are parallel the field.

The force on AB which is perpendicular to B is directed into the plane of the loop given by,

$F_1 = IlB\sin\theta = IbB$ ……….. $\theta = 90^0$



Similarly the force on CD is directed out of the plane of the paper,

$F_2 = IbB = F_1$

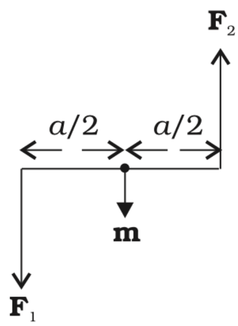
Thus the net force is zero.

This pair of forces exerts a torque on the loop which tends to rotate the loop. It is given by,

$\tau = F_1\left(\frac{a}{2}\right) + F_2\left(\frac{a}{2}\right)$………… basically torque = force x ⊥distance

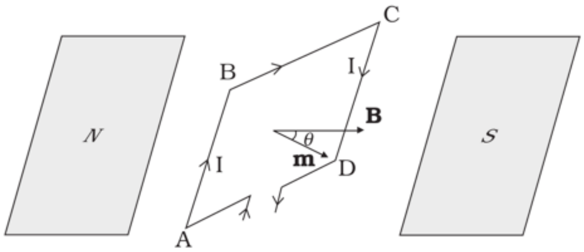
i.e, $\tau = IbB\left(\frac{a}{2}\right) + IbB\left(\frac{a}{2}\right) = I(ab)B$

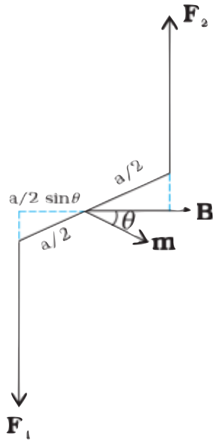
$\therefore \tau = IAB = mB$ ……….(1)



where A=ab is the area of the loop and m=IA is the magnetic dipole moment of loop.

When the plane of the loop is not along the magnetic field and makes an angle with it,





Let the angle between the field B and normal to the coil is θ.
The equal and opposite forces on AB and CD are such that $F_1 = F_2 = IbB$ (non collinear)
These two non-collinear forces constitutes a couple and the torque is,

$$\tau = F_1\left(\frac{a}{2}\sin\theta\right) + F_2\left(\frac{a}{2}\sin\theta\right)$$ where the perpendicular distance is $\frac{a}{2}\sin\theta$

$$\implies \tau = IAB\sin\theta = mB\sin\theta$$ ....(2)

As $\theta \to 0$, the $\perp$distance between the forces also approaches zero and collinear forces which results in zero net force and zero torque.

In general, $\vec{\tau} = \vec{m} \times \vec{B}$
this is similar to $\vec{\tau} = \vec{p}_e \times \vec{E}$ in case of electric dipole of of dipole in electric field.

From the above, it can be seen that any small rotation of the coil produces a torque which brings it back to its original position.

Current Loop as a Magnetic Dipole

Magnetic field on the axis of a circular loop of radius R carrying current I is directed along the axis given by Right Hand Thumb Rule. It is given by,

$$B = \frac{\mu_0 I\, R^2}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

At very large distances when $x \gg R$, we can drop R in the denominator and hence

$$B = \frac{\mu_0 I\, R^2}{2x^3} = \frac{\mu_0 I\, A}{2\pi x^3}$$ since Area, $A = \pi R^2$

But m=IA

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3}$$

Substituting the below, we get,

$\mu_0 \rightarrow \frac{1}{\epsilon_0}, \vec{m} \rightarrow \vec{P}_e, \vec{B} \rightarrow \vec{E}$

$\implies \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}_e}{x^3}$ this is precisely the field of electric dipole on its axis.

NOTE:

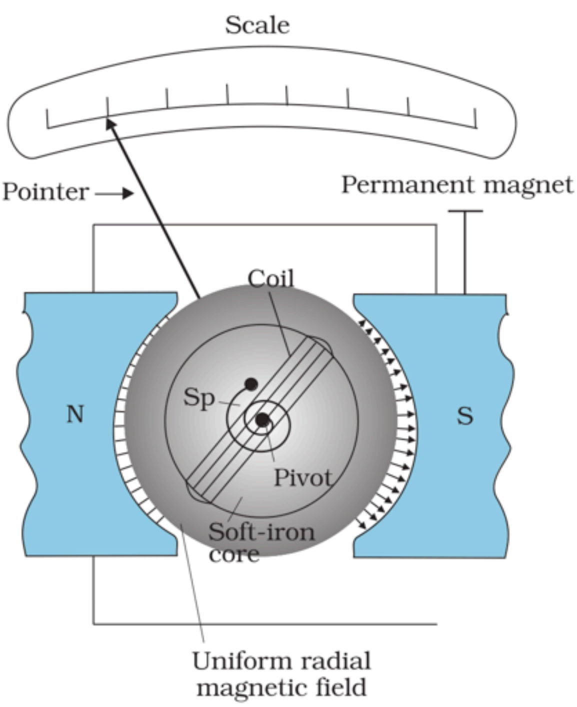
With the above analogy the electric field on the perpendicular bisector of the dipole which is $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}_e}{x^3}$ **gives us the magnetic field in the plane of the loop to be** $\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{m}}{x^3}$

The Moving Coil Galvanometer (MCG)

MCG is a a device used to detect / measure very currents. It consists of a coil of area A and number of turns N free to rotate about a fixed axis suspended in uniform radial magnetic field, B. A cylindrical soft iron core helps the field to be strong and radial.

If I is the current through the coil, the torque acting on it is given by,

$\tau = mB = nIAB$ …………. Nm-1



Since the field is radial, the plane of the coil is always parallel to the field and $\theta = 90^0$

It tends to rotate the coil.

The spring Sp provides a counter torque τ_{res} which balances the applied torque τ giving us a steady deflection ϕ .

i.e., $\tau_{res} \propto \phi$

$\implies \tau_{res} = k\phi$

Here ϕ is the steady angular and k is the proportionality constant called torsional constant also called the restoring torque per unit twist.

Restoring torque = Applied Torque at equilibrium.

i.e, $\tau_{res} = \tau$

$$\implies k\phi = NIAB$$

$$\implies \phi = \left(\frac{NAB}{k}\right) I$$

$\implies \phi \propto I$ Where $\left(\frac{NAB}{k}\right)$ remains a constant.

The ratio $\frac{\phi}{I} = \frac{NAB}{k}$ is called current sensitivity of the galvanometer (it is the deflection per unit current)

We can increase the sensitivity by just increasing the number of turns.

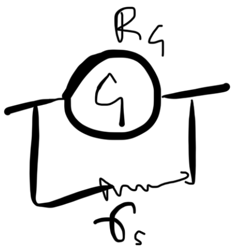
Conversion of Galvanometer into an Ammeter:

Galvanometer shows enough deflection even for a small current and hence gives full deflection for currents of μA and can not be used to measure larger currents.

Ammeter is a device used to measure currents. Hence it is required to be connected in series in a circuit.

If a galvanometer is used in series in a circuit, the current gets altered due to the high resistance of galvanometer.

To overcome this, a very small resistance is connected in parallel (called shunt) to the galvanometer to convert it into an Ammeter.



The effective resistance of the converted Ammeter then becomes,

$$R_{eff} = \frac{R_G r_s}{R_G + r_s} = r_s \quad \text{if } R_G >> r_s$$

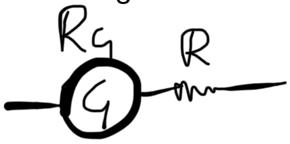
Thus the small resistance r_s in series in any circuit may not alter the actual current effectively.

Conversion of Galvanometer into a Voltmeter:

Voltmeter is a device used to measure Voltage (Potential Difference) across two points in any circuit.

It is always connected in parallel in a circuit and hence should have high resistance to avoid excess flow of the actual current through the voltmeter itself.

A high resistance is connected in series with a galvanometer to convert it into a voltmeter.



The effective resistance of the converted Voltmeter then becomes,

$R_{eff} = R_G + R = R$ if $R >> R_G$

Also, like current sensitivity, voltage sensitivity is defined as the deflection per unit voltage. i.e.,

$$\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\left(\frac{I}{V}\right) = \left(\frac{NAB}{k}\right)\left(\frac{1}{R}\right)$$

Resistance R varies directly with number of turns N and hence increase in N does not increase the voltage sensitivity.

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FORMULAE

Magnetic Force:
$F_B = q[\vec{v} \times \vec{B}] = qvB\sin\theta\ \hat{n}$

Mg. Force on St. Conductor in B
$F = q(\vec{V} \times \vec{B}) = (nlA)e(\vec{v_d} \times \vec{B}) = I(\vec{l} \times \vec{B})$

Radius of charge path in B
$r = \frac{mv}{qB}$

Period of charge rotation in B
$T = \frac{1}{f_c} = \frac{2\pi m}{qB}$

Pitch of helical path
$p = v_{||}T = v_B(\frac{2\pi m}{qB})$

Velocity Selector
$v = \frac{E}{B}$

B on axis of circular coil
$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$

B at the centre of the coil
$B = \frac{\mu_0 I}{2R}$

Ampere's Circuital Law
$\oint \vec{B} \cdot \vec{dl} = \mu_0\ I_e$

B of long straight conductor
$B = \frac{\mu_0\ I}{2\pi r}$

Solenoid
$B = \mu_0\ nI$

B at the end of solenoid.
$B = \frac{\mu_0\ nI}{2}$

Force between parallel conductors
$f_{ba} = \frac{F_{ba}}{L} = \frac{\mu_0 I_a I_b}{2\pi d}$

Magnetic Moment of current loop
$\vec{m} = NI\vec{A}$

Torque on a current loop
$\tau = IAB\sin\theta = mB\sin\theta$

Current loop as a magnetic Dipole
$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3}$

MCG
$\frac{\phi}{I} = \frac{NAB}{k}$

Eff. Resistance of Converted Ammeter
$R = \frac{R_G r_s}{R_G + r_s} = r_s$

Eff. Resistance of Converted Voltmeter.
$R_{eff} = R_G + R = R$